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SUB CODE : 041 MEDIUM : E

(BVETVAEZVSABSB)

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D.G.E -HR.SEC. EXAMINATION MARCH - 2014

REGISTER NUMBER **606676**

CANDIDATE NAME : AKALYA M

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GROUP CODE : 103

SUBJECT : **041 MATHEMATICS E**

APPLICATION NO : 1010636

APPLIED FOR : ScanCopy

(B)

D.G.E -HR.SEC. EXAMINATION MARCH - 2014

(BVETVAEZVSABSB)

SUBJECT :

041 MATHEMATICS E

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Marks in Words			Marks in Figures		

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அரசுத் தேர்வுகள் துறை

DEPARTMENT OF GOVERNMENT EXAMINATIONS

Script No. 02

Total Marks 196

HSE

விடைத்தாள் திருத்துவோர் நிறைவு செய்ய வேண்டியவை

FOR THE USE OF EXAMINERS ONLY

வினாவாரியாக மொத்தம் Questionwise Total										பக்கவாரியாக மொத்தம் Pagewise Total			
வினா எண் Q.No	மதிப்பெண்கள் Marks	வினா எண் Q.No	மதிப்பெண்கள் Marks	வினா எண் Q.No	மதிப்பெண்கள் Marks	வினா எண் Q.No	மதிப்பெண்கள் Marks	வினா எண் Q.No	மதிப்பெண்கள் Marks	பக்க எண் Page No	மதிப்பெண்கள் Marks	பக்க எண் Page No	மதிப்பெண்கள் Marks
1	1	23	1	45		67		89		1	12	23	
2	1	24	0	46	6	68	10	90		2	26	24	
3	1	25	1	47	6	69		91		3	10	25	
4	1	26	1	48		70	10	92		4	10	26	
5	1	27	1	49	6	71		93		5	10	27	
6	1	28	1	50	5	72		94		6	10	28	
7	1	29	1	51	6	73		95		7	10	29	
8	1	30	1	52	6	74		96		8	10	30	
9	1	31	1	53		75		97		9	10	31	
10	1	32	1	54		76		98		10	10	32	
11	1	33	1	55	6	77		99		11	10	33	
12	1	34	1	56		78		100		12	10	34	
13	1	35	1	57	10	79		101		13	6	35	
14	0	36	1	58	10	80		102		14	5	36	
15	1	37	1	59		81		103		15	12	37	
16	1	38	1	60	10	82		104		16	11	38	
17	1	39	1	61	10	83		105		17	12	39	
18	1	40	1	62	10	84		106		18	6	40	
19	1	41	1	63	10	85		107		19	6	41	
20	1	42	6	64	10	86		108		20		42	
21	1	43	6	65	10	87		109		21		43	
22	1	44	5	66		88		110		22		44	
மொத்தம் Total	21	மொத்தம் Total	34	மொத்தம் Total	121	மொத்தம் Total	20	மொத்தம் Total		மொத்தம் Total	196	மொத்தம் Total	

வினாவாரியாக ஒட்டு மொத்தம்
Question-wise Grand Total: 196

பக்கவாரியாக மொத்தம்
Page-wise Total: 196

தேர்வு எழுதுபவர் செய்யக்கூடியவை மற்றும் செய்யக்கூடாதவை

Do's & Don'ts for Candidates

1. முகப்புச்சீட்டில் உரிய இடத்தில் கையொப்பமிட வேண்டும்.

   Put your signature in the Top sheet in the appropriate place.

2. விடைத்தாளில் ஒரு பக்கத்திற்கு 20 முதல் 25 வரிகள் வரை எழுதவேண்டும்.

   Write 20 to 25 lines in a page.

3. விடைத்தாளின் இருபுறத்திலும் எழுத வேண்டும்.

   Write answers in both sides of paper.

4. செய்முறைகள் யாவும் விடைத்தாளின் கீழ் பகுதியில் இடம் பெறவேண்டும்.

   All rough works must be done on the lower part of the page.

5. வினா எண் தவறாமல் எழுத வேண்டும்.

   Write the question numbers without fail.

6. இரு விடைகளுக்கிடையே இடைவெளி விட்டு எழுத வேண்டும்.

   Leave space between two answers.

7. வினாத்தாளின் வரிசை (A or B) எழுத வேண்டும்.

   Write the question paper booklet series. (A or B)

8. விடைத்தாளில் நீலம்/கருப்புமை கொண்ட பேனாவால் விடைகளை தெளிவாக எழுத வேண்டும்.

   Anwers must be legibly written either in Blue or Black ink pen.

9. விடைத்தாளில் எழுதாத பக்கங்களில் குறுக்குக்கோடு இடவேண்டும்.

   Cross the unwritten pages.

1. வினாத்தாளில் எந்தவித குறியீடும் இடக்கூடாது.

   No marking in the question paper.

2. விடைத்தாளை சேதப்படுத்தக் கூடாது.

   Don't damage the answer paper.

3. விடைத்தாளில் எந்த ஒரு பக்கத்திலும் தேர்வு எண்/பெயர் எழுதக்கூடாது.

   Don't write name / Register Number in any page of the answer book.

4. வண்ணக்கலர் கொண்ட பேனா/ பென்சில் எதையும் பயன்படுத்தக் கூடாது.

   Don't write with sketch colour pencils.

5. விடைத்தாள் கோட்டின் இடது பக்கத்தில் எழுதக்கூடாது.

   Don't write on the margin.

6. விடைத்தாள் புத்தகத்தின் எந்த தாளையும் கிழிக்கவோ/நீக்கவோ கூடாது.

   Don't tare / remove any page from the answer book.

DGE HSE

PART - A

1. 1) $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$

2. 3) $\frac{1}{k} I$

3. 1) 1

4. 1) only trivial solution

5. 3) $a = \frac{1}{|m|}$

6. 4) $\pi/2$

7. 4) 60°

8. 1) $[\vec{r} - \vec{a}, \vec{b} - \vec{a} \quad \vec{c} - \vec{a}] = 0$

9. 1) $(0, 0, -4)$

10. 3) $(3, -4, 5), 7$

11. 1) purely imaginary

12. 4) fourth quadrant

12

$(-6\lambda+6, \ 4\lambda-4, \ -8\lambda+4)$ $(2\mu-1, 4\lambda-2, -2\lambda-3)$

$\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$ $\rho(A)$

-2μ $(3, -4, 5)$

$\cos\pi/3 + i\sin$ $0 + c$

G.Ex. 54–1A

$-6\lambda + 6 = 2\mu - 1$ $\quad 2\mu + 6\lambda = 7$ $\quad (\vec{a} + \vec{b})^2 = (-\vec{c})^2$

$\frac{7}{8}$ $\quad 4\lambda - 4 = 4\mu - 2$ $\quad 8\lambda = -7$ $\quad 4\mu + 12\lambda = 14$ $\quad |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = c^2$

$4\lambda - 4\mu = 2$ $\quad \lambda = 7/8$ $\quad -4\mu + 4\lambda$ $\quad \cos\theta = 09 + 16 + 2 \times 12 \times \cos\theta = 25$

13. 3) $-2i\sin(\alpha-\beta+\gamma)$

14. 3) $-z$ ~~is~~ the point symmetrical to z about origin

15. 3) 4

16. 3) 8

17. 2) 4

18. 2) $(a\,t_1 t_2,\ a(t_1+t_2))$

19. 3) ~~30~~ $27t+80$.

20. 3) $\pi/2$

21. 3) $3a+b=0$

22. 2) $f(a+h) = f(a)+hf'(a+\theta h)$ $0<\theta<1$

23. 3) $\cos\theta$

24. 1) $x \geq a$

25. 4) $3\pi/8$

$9x^2+5y^2=180$

$\frac{x^2}{5}$

$x=\frac{t}{2}$

$2dx=dt$

$\frac{1}{2}\int \sin^4\frac{t}{2}\,dt$

$\frac{1}{2}\times\frac{3}{4}\cdot\frac{1}{2}\frac{\pi}{2}$

$\frac{3\pi}{32}$

T, F, $TVT=T$, $T\wedge T=T$.

$\frac{1}{2}\int_0^{\pi/2}\sin^4\frac{t}{2}\,dt$

$\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}$

$y^2-4x+4y+8=0$

$y^2+4x=4x-8$

$y^2+4x+4-4=4x-8+4$

$(y+2)^2=4x-4$

$(y+2)^2=4(x-1)$

$Y^2=4X$

$a=1$

$x=\frac{t}{2}$, $2x=t$, $dx=\frac{dt}{2}$

$xy=8$

$xy=\frac{a^2}{2}$

$\frac{a^2}{2}=8$

$a^2=16$

$e=\sqrt{1-\frac{20}{36}}=\sqrt{\frac{16}{36}}=\frac{4}{6}$

$4a=4a=4$

$\frac{x^2}{20}+\frac{y^2}{36}=1$

$a^2=36$, $b^2=20$

$4a=4$

$a=1$

$xy=\frac{a^2}{2}$

$\frac{a^2}{2}=8$

$a^2=16$

$a=4$

$ae=\frac{4}{6}\times 6$

3

26. 4) ~~$2^7 \lfloor 6$~~ 2) $\frac{1}{3}$

27 ~~2) $\frac{1}{3}$~~ 4) 9π

28. 4) $2^7 \lfloor 6$

29 2) $x = ce^{-my}$

30 1) $(y')^2 - xy' + y = 0$

31 2) $\cos x$

32 4) 1, 3

15

33 3) (i), (iii), (iv)

$a*b = a+b-1$
$a*e = a+e-1$
$a+e-1 = a$
$e = 1$

34. 4) $p \wedge (\sim p)$

35. 2) 1

36. 4) $p \leftrightarrow q$ is true.

37. 3) 4

38 1) 16

39. 3) $\frac{2}{3}$

40. 3) $P\{X = x_n\} = F(x_n) - F(x_{n-1})$

T F
F T
F F
$a*$

$\left(\frac{dy}{dx}\right)^3$

$e^{\int \tan x}$
$e^{\log \sec^2 x}$
$\frac{n-1}{1}$ $e^{\log \sec}$
$3-1$

$e^{\int m\,dy}$
$x e^{my}$

$e^{\int P dx}$
$e^{\int P dy}$

G.Ex. 54-2

x	0
t	$\pi/2$

$2x = t$
$x = \frac{t}{2}$
$2 = \frac{dt}{dx}$

$\frac{1}{2}\int_0^{\pi/2} \cos^3 t \, dt$ $2x\,dx$

$\frac{1}{2} \times \frac{2}{3}$ $\frac{1}{3}$

$\frac{6!}{\left(\frac{1}{2}\right)^7}$
2^7

$\frac{1}{3}\pi r^2 h$
$\frac{1}{3}\pi \times 9 \times 9$
$\frac{27}{3}$

$m\,dy$
my
$xe^{my} = c$

PART - C.

57. The required plane contains the line

$$\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{3}$$

The required plane passes through the point $(2,2,1)$ and $\parallel^{l}$ to

$\vec{u} = 2\vec{i}+3\vec{j}+3\vec{k}$.

The required plane is parallel to

$\vec{v} = 3\vec{i}+2\vec{j}+\vec{k}$

The required plane passes through one point and $\parallel^{l}$ to 2 vectors.

Vector equation:

The equation of plane passing through point $(\vec{a})$ and parallel to two vectors $\vec{u}$ & $\vec{v}$ is given by

$$\vec{r} = \vec{a} + s\vec{u} + t\vec{v}$$

$\vec{a} = 2\vec{i} + 2\vec{j} + \vec{k}$

$\vec{u} = 2\vec{i} + 3\vec{j} + 3\vec{k}$

$\vec{v} = 3\vec{i} + 2\vec{j} + \vec{k}$

$$\vec{r} = 2\vec{i} + 2\vec{j} + \vec{k} + s\,(2\vec{i} + 3\vec{j} + 3\vec{k}) + t\,(3\vec{i} + 2\vec{j} + \vec{k})$$

Cartesian equation :

The required plane in cartesian plane is,

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

Here $(x_1, y_1, z_1) = (2, 2, 1)$

$(l_1, m_1, n_1) = (2, 3, 3)$

$(l_2, m_2, n_2) = (3, 2, 1)$

$$\begin{vmatrix} x - 2 & y - 2 & z - 1 \\ 2 & 3 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$(x-2)(3-6) - (y-2)(2-9) + (z-1)(4-9) = 0$

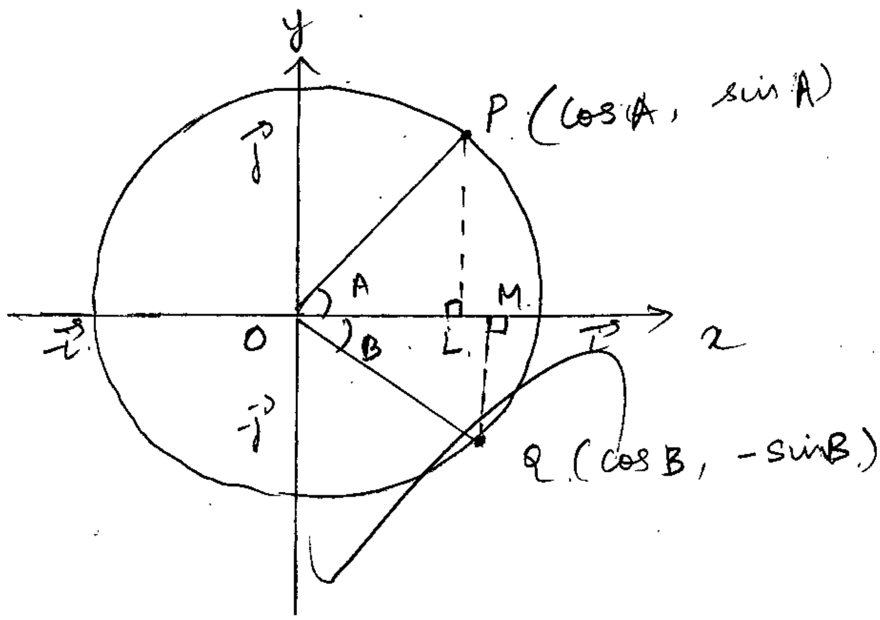
$(x-2)(-3) - (y-2)(-7) + (z-1)(-5) = 0$

$$-3x+6+7y-14-5z+5=0.$$

$$-3x+7y-5z-3=0.$$

$$\boxed{3x-7y+5z+3=0}.$$

58. <u>To prove</u>:
$\cos(A+B) = \cos A \cos B - \sin A \sin B$



(i) Let us consider a unit circle of radius 1 unit with centre O.

(ii) Let P, and Q be two points on the unit circle making angles A and B with +ve X-axis such that,

$\angle POQ = \angle POX + \angle QOX$

$\angle POX = A + B$

(iii) Let PL and QM be the two perpendiculars drawn from P and Q respectively to X - axis.

(iv) Let the coordinates of P (cosA, sinA) and Q (cosB, -sinB)

(v) Let $\vec{i}$, $\vec{j}$ be the unit vectors acting along x axis and y-axis respectively.

In rt. Δ OLP.

By triangle law,

$\vec{OP} = \vec{OL} + \vec{LP}$

$\vec{OP} = \cos A\,\vec{i} + \sin A\,\vec{j} \longrightarrow (1)$

10

In rt $\triangle$ OMQ

$$\vec{OQ} = \vec{OM} + \vec{MQ}$$

$$= \cos B\,\vec{i} + \sin B\,(-\vec{j})$$

$$= \cos B\,\vec{i} - \sin B\,\vec{j} \rightarrow ②$$

$$\vec{OP}\cdot\vec{OQ} = (\cos A\,\vec{i} + \sin A\,\vec{j})\cdot(\cos B\,\vec{i} - \sin B\,\vec{j})$$

$$\vec{OP}\cdot\vec{OQ} = \cos A\cos B - \sin A\sin B \rightarrow ③$$

By definition of dot product,

$$\vec{OP}\cdot\vec{OQ} = |\vec{OP}|\,|\vec{OQ}|\cos(A+B)$$

$$\vec{OP}\cdot\vec{OQ} = (1)\,(1)\cos(A+B) \rightarrow ④$$

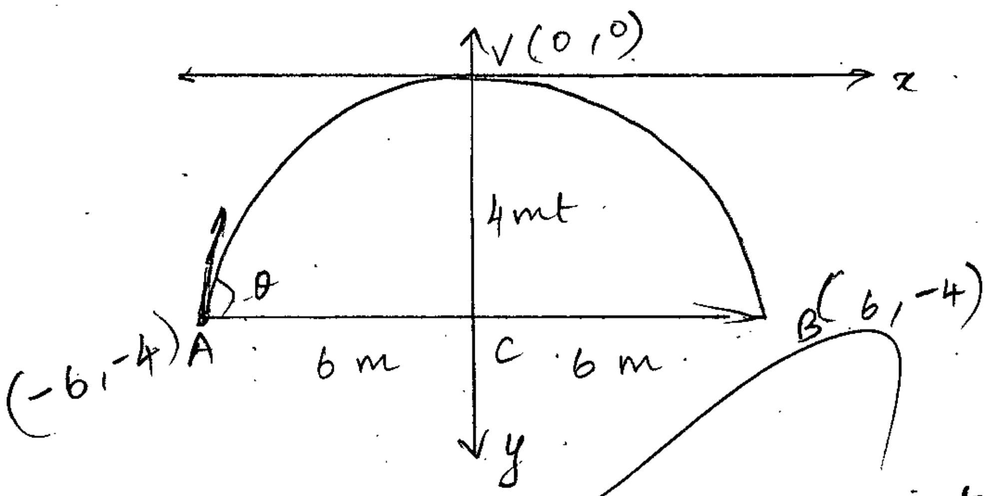
From ③ & ④

③ = ④

$$\cos(A+B) = \cos A\cos B - \sin A\sin B.$$

Hence proved.

60)



Let the cracker be projected at point A and AB shows the parabolic path.

<u>Given</u>: The maximum height reached is 4 mt.

Let the <u>vertex</u> be at the <u>origin</u>

$VC = 4\ m$

$AC = 6\ m = BC$

The parabola is open down.

Eqn: of parabola open down with vertex at origin is

$$x^2 = -4ay \longrightarrow (1)$$

As the point $A(-6,-4)$ passes through the parabola,

$$(-6)^2 = -4(a)(-4)$$

$$36 = 16a.$$

$$a = \frac{36}{16} = \frac{9}{4}$$

$$\boxed{a = \frac{9}{4}}$$

$\therefore$ Eqn of parabola is $x^2 = -9y \rightarrow (1)$

Diff (1) w.r.t x:

$$2x = -9\frac{dy}{dx}.$$

$$\frac{dy}{dx} = \frac{-2x}{9}.$$

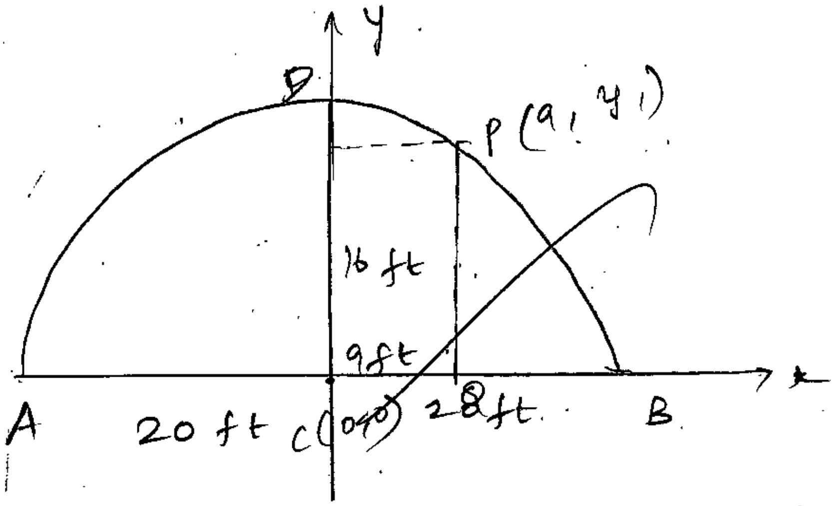
$$\tan\theta = \frac{dy}{dx}_{(-6,-4)} = \frac{-2\times-6}{9}.$$

$$\tan\theta = \frac{12}{9}.$$

$$\tan\theta = \frac{4}{3}.$$

The Angle of projection $= \boxed{\theta = \tan^{-1}\frac{4}{3}}$

61)



Let AB represent the arch of the bridge in the form of semi-ellipse.

Let the centre be at the origin.

The span of bridge is 40 m.

Here, CA = CB = 20ft

The height of bridge = 16ft.

To find: PQ ie. the height of the arch 9 ft from the centre.

Equ of ellipse is,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Here $a = 20$, $a^2 = 20^2$

$b = 16$, $b^2 = 16^2$

$$\frac{x^2}{20^2} + \frac{y^2}{16^2} = 1$$

The point $P(9, y_1)$ also passes through the ellipse,

$$\therefore \frac{9 \times 9}{20 \times 20} + \frac{y_1^2}{16^2} = 1$$

$$\frac{y_1^2}{16 \times 16} = 1 - \frac{81}{400}$$

$$\frac{y_1^2}{16^2} = \frac{319}{400}$$

$$\frac{y_1^2}{16^2} = \frac{319}{400}$$

$$y_1^2 = \frac{16 \times 16 \times 319}{20 \times 20}$$

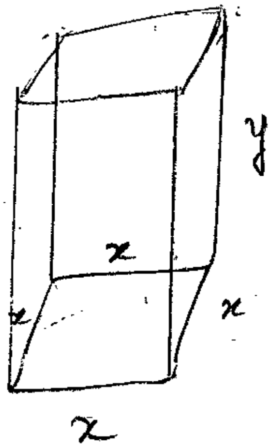
$$y_1 = \frac{16}{20}\sqrt{319}$$

$$y_1 = \frac{4}{5}\sqrt{319} \text{ ft}$$

$$\begin{array}{r} 400 \\ 81 \\ \hline 319 \end{array}$$

$\therefore$ The height of the arch 9 ft from the centre is $\frac{4}{5}\sqrt{319}$ ft

63.



Given:

Volume of cuboid = 2000 cc.

Let x, y be the dimensions of the cuboid.

To minimize the dimensions of box:

Area of the 2 square bases = $2x^2$

Cost per sq. m at the top and bottom = 3 ₹

Total cost of the bottom & top areas = $2x^2 \times 3$

$= 6x^2$

Cost of the sides per sqm = 1.50

Area of the sides (4) = $4xy$

Total cost of the sides = $4xy \times 1.50$

$= 6xy$

Total cost of the cuboid box $= 6x^2 + 6xy$

Given: Volume $= 2000$.

$$x^2 y = 2000.$$

$$y = \frac{2000}{x^2}.$$

To minimize the cost.

$$A = 6x^2 + 6x \times \frac{2000}{x^2}.$$

$$A(x) = 6x^2 + \frac{12000}{x}.$$

$$A'(x) = 12x + \frac{12000}{x^2}(-1)$$

$$= 12x - \frac{12000}{x^2}.$$

For max, min $A'(x) = 0$,

$$12x - \frac{12000}{x^2} = 0.$$

$$12x^3 - 12000 = 0.$$

$$\Rightarrow x^3 = \frac{12000}{12}.$$

$$x^3 = 1000 \Rightarrow x = 10 \text{ cm}$$

$$A''(x) = 12 + \frac{2(12000)}{x^3}$$

$A''(10) = +ve$

$\therefore$ The cost is minimum when $x = 10$ cm.

$\therefore$ When $x = 10$,

$$y = \frac{2000}{x^2}$$

$$= \frac{2000}{10 \times 10}$$

$$= \underline{20\text{ cm}}$$

$\therefore$ The length and breath of the base (square) is $\underline{10\text{ cm}}$.

Height of the cuboid $= \underline{20\text{ cm}}$.

64. $$f(x, y) = \frac{1}{\sqrt{x^2+y^2}}.$$

10

$$f(tx, ty) = \frac{1}{\sqrt{t^2x^2+t^2y^2}}$$

$$= \frac{1}{t\sqrt{x^2+y^2}}$$

$\sqrt{(tx)^2}$

$t^2(x^2+y^2)$

$$= \frac{t^{-1}}{\sqrt{x^2+y^2}}.$$

$$= t^{-1} f(x, y)$$

$\therefore$ The degree of t in homogenous function, $n = -1$.

By Euler's theorem,

$$\boxed{x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf.} \rightarrow (1)$$

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = -f \rightarrow (1)$$

<u>To verify Euler's theorem:</u>

$$(x^2+y^2)^{-1/2 - 1} \quad -1 \; (x^2+y^2)^{\frac{-1}{2}}$$

$$\frac{\partial f}{\partial x} = -\frac{1}{\cancel{2}(x^2+y^2)^{3/2}} \times \cancel{2}x$$

$$-\frac{1}{\sqrt{x^2+y^2}} \quad x^2+y^2$$

$$x\frac{\partial f}{\partial x} = -\frac{x}{\cancel{2}(x^2+y^2)^{3/2}} \times \cancel{2}x \longrightarrow (2) \qquad -\frac{1}{2}$$

$$\frac{\partial f}{\partial y} = -\frac{1}{\cancel{2}(x^2+y^2)^{3/2}} \times \cancel{2}y$$

$$y\frac{\partial f}{\partial y} = -\frac{\cancel{2}y^2}{\cancel{2}(x^2+y^2)^{3/2}} \longrightarrow (3)$$

Adding (2) & (3),

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = -\frac{x^2}{(x^2+y^2)^{3/2}} - \frac{y^2}{(x^2+y^2)^{3/2}}$$

$$= -\left(\frac{x^2+y^2}{(x^2+y^2)^{3/2}}\right)$$

$$= -\left(\frac{1}{(x^2+y^2)^{1/2}}\right)$$

$$= -\left(\frac{1}{\sqrt{x^2+y^2}}\right)$$

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = -f(x,y) \longrightarrow (4)$$

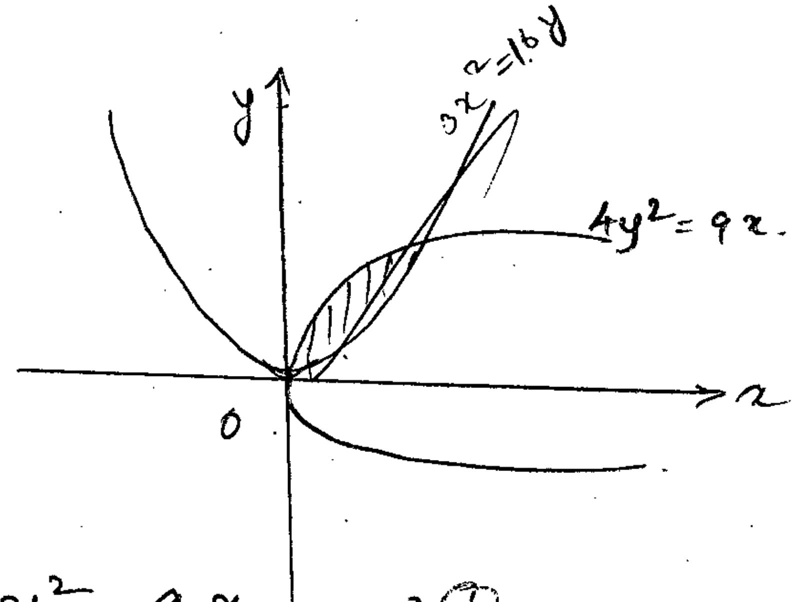
10

From ① & ④

① = ④

∴ Hence Euler's theorem is verified.

65.



$4y^2 = 9x \rightarrow$ ①

$3x^2 = 16y \rightarrow$ ②

solving ① & ②

$4\left(\frac{3x^2}{16}\right)^2 = 9x$

$\frac{9 \times x^4}{64} = 9x$

$x^4 - 64x = 0$

$x(x^3 - 64) = 0$

$\Rightarrow \quad x = 0, \quad x = 4.$

when $x = 0, \quad y = 0.$

$x = 4, \quad y = 3.$

$\frac{3 \times \cancel{4} \times 4}{16}$

(0,0) (4,3) are the points of intersection of the parabolas

Area common to the parabolas $= \int (x_1 - x_2)\, dy.$

Lts : $y = 0, \quad y = 3$

$$\frac{4 \times 2 y^{3/2}}{\sqrt{3}\; 3} \quad \frac{8}{3\sqrt{3}} \left[3^{3/2}\right]$$

$$\text{Area} = \int_0^3 \frac{4y^2}{9} - \sqrt{\frac{16y}{3}}\; dy.$$

$$= \int_0^3 \frac{4y^3}{9} dy - \int_0^3 \frac{4}{\sqrt{3}} y^{1/2}\, dy.$$

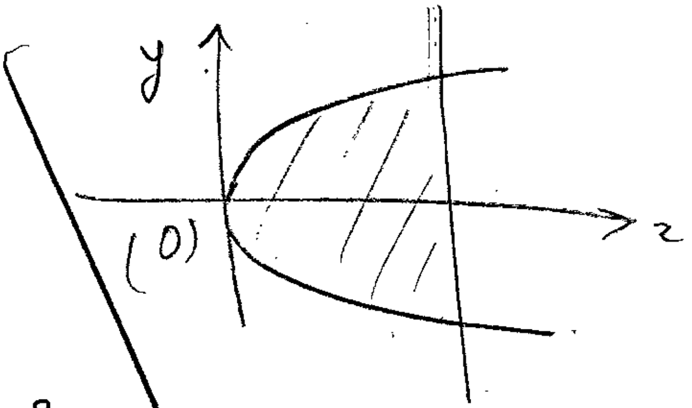
$$= \frac{4}{9}\left[\frac{y^4}{4}\right]_0^3 - \frac{4}{\sqrt{3}}\left[\frac{2 \times y^{3/2}}{3}\right]_0^3$$

$$= \frac{\cancel{4}}{\cancel{4} \times 9}\left[3^4 - 0\right] - \frac{4 \times 2}{\sqrt{3} \times 3}\left[3^{3/2} - 0\right]$$

$$= \frac{3^4}{9} - \frac{8 \times \cancel{3}\sqrt{\cancel{3}}}{\cancel{3}\sqrt{\cancel{3}}}$$

$$= \frac{27}{9} - 8 \times 3 = 27 - 24 = 4 \text{ sq. units}$$

66.



$$y^2 = 4ax \longrightarrow (1)$$

Latus rectum $=$ $x = a$.

Surface Area by revolving $y^2 = 4ax$ about x-axis $= \int 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

Lts: $x = 0$, $x = a$.

Diff (1) w.r.t x:

$$2y\frac{dy}{dx} = 4a$$

$$\frac{dy}{dx} = \frac{4a}{2y}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{4a^2}{y^2} = \cancel{\frac{4a^2}{4 \times 4a^2x^2}}$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1 + \frac{4a^2}{4ax}$$

$$1+\left(\frac{dy}{dx}\right)^2 = \frac{x+a}{x}$$

$2y\frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{4a}{2y}$

$= \frac{2a}{y} \cdot \frac{2a}{y^2}$

$\frac{4a^2}{y^2}$

$\frac{4a^2}{4a\,x}$

81

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{\frac{x+a}{x}}$$

$$= \sqrt{\frac{x+a}{x}}$$

$2\pi\sqrt{4ax}\int \frac{x+a}{x}$

$2\pi\sqrt{4a}\int \sqrt{x}\ \frac{x+a}{\sqrt{x}}$

$4\pi a \int \sqrt{x+a}$

$\left[\frac{1}{2}\frac{1}{\sqrt{x+a}}\right]_0^a$

<u>Surface area</u> : $= \int_0^a 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

Lts $x = 0$
$x = a$

$\frac{1}{2}\left(\frac{1}{\sqrt{2a}} - \frac{1}{\sqrt{a}}\right)$

$$= \int_0^a 2\pi\sqrt{4ax}\ \sqrt{\frac{x+a}{x}}\, dx$$

$\left(\frac{1}{\sqrt{2a}} - \frac{1}{\sqrt{a}}\right)$

$4\pi\sqrt{a}\times\frac{1}{\sqrt{a}}\,\frac{1}{2}\left(\frac{1}{\sqrt{2}} - 1\right)$

$$= 4\pi\sqrt{a}\int_0^a \cancel{\sqrt{x}}\ \frac{\sqrt{x+a}}{\cancel{\sqrt{x}}}$$

$$= 4\pi\sqrt{a}\int_0^a \sqrt{x+a}$$

$(x+a)^{1/2}$

$\frac{1}{2}(x+a)^{-1/2}$

$$= 4\pi\sqrt{a}\left(\frac{+1}{2\sqrt{x+a}}\right)_0^a$$

$$= 4\pi\sqrt{a}\left[+\frac{1}{2\sqrt{a+a}} - \left(\frac{+1}{2\sqrt{a}}\right)\right]$$

$$= 4\pi\sqrt{a}\left[+\frac{1}{2\sqrt{2}\sqrt{a}} - \frac{1}{2\sqrt{a}}\right]$$

$$= \frac{4\pi\sqrt{a}}{2\sqrt{a}}\left[-\frac{1}{\sqrt{2}} - \right.$$

62 $x^2 - 3y^2 + 6x + 6y + 18 = 0$

0

$x^2 + 6x - 3y^2 + 6y = -18$

$(x^2 + 6x + 9 - 9) - 3(y^2 - 2y) = -18$

$(x^2 + 6x + 9 - 9) - 3(y^2 - 2y + 1 - 1) = -18$

$(x+3)^2 - 3(y-1)^2 = -18 + 9 - 3$

$(x+3)^2 - 3(y-1)^2 = -12$

$-\frac{(x+3)^2}{12} - \frac{3(y-1)^2}{-12} = 1$

$\frac{(y-1)^2}{4} - \frac{(x+3)^2}{12} = 1$

Let $X = x+3$, $Y = y-1$

$\frac{Y^2}{4} - \frac{X^2}{12} = 1$

Here ; $a^2 = 4$. $a = 2$

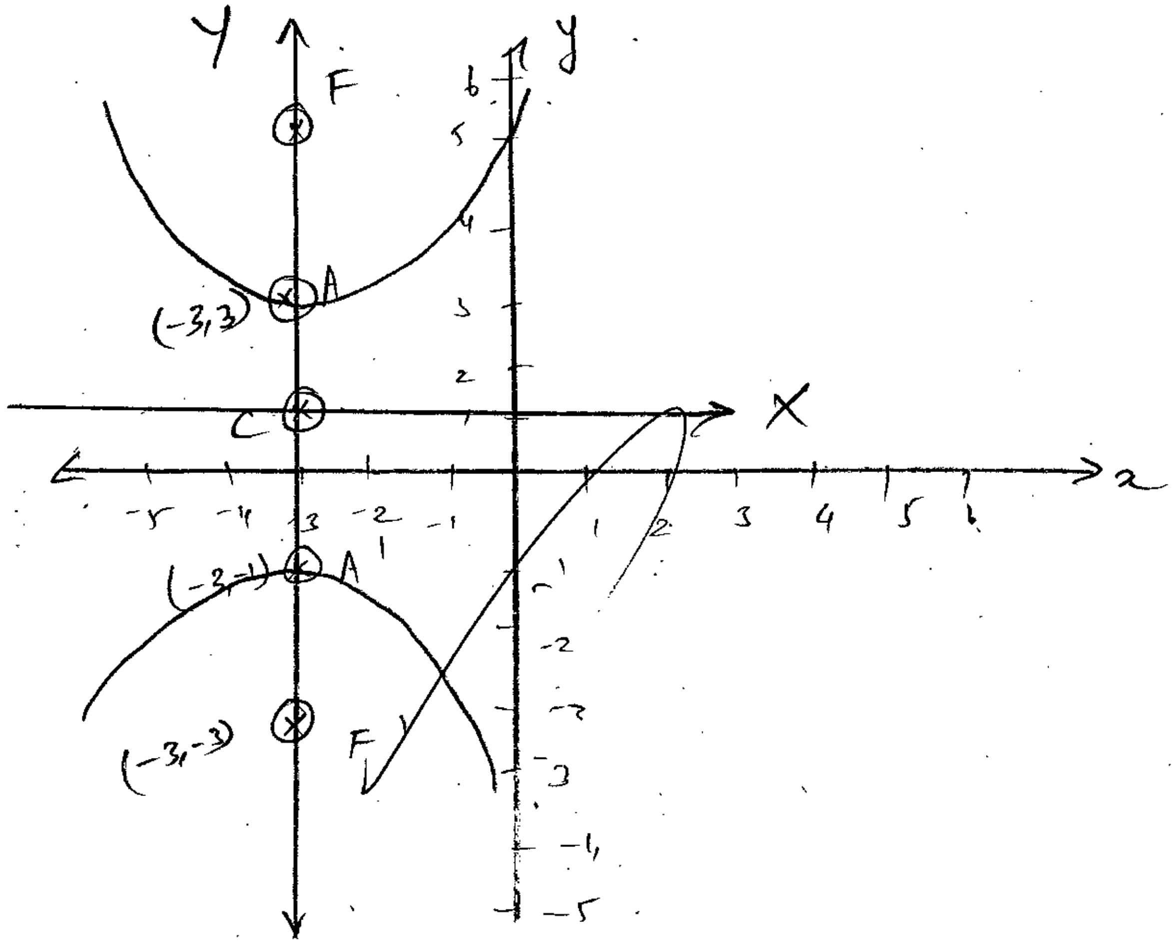
$b^2 = 12$. $b = \sqrt{12}$

The transverse axis along Y-axis

$$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{12}{4}} = \sqrt{\frac{16}{4}}$$
$$= \frac{4}{2} = 2.$$

$$ae = 2 \times 2 = 4.$$

	with refrence X, Y axis	with refrence to x and y axis. $X = x+3$, $Y = y-1$.
Centre	$C(0,0)$	$0 = x+3 \Rightarrow x = -3.$ $0 = y-1$ $y = 1.$ $C(-3,1)$
Vertices	$F(0, \pm a)$ $(0, \pm 2)$	$0 = x+3 \Rightarrow x = -3.$ $2 = y-1 \Rightarrow y = 3.$ $-2 = y-1 \Rightarrow y = -1.$ $A(-3,3)$, $A'(-3,-1)$
Focci	$A(0, \pm ae)$ $A(0, \pm 4)$	$0 = x+3 \Rightarrow x = -3.$ $4 = y-1 \Rightarrow y = 5.$ $-4 = y-1 \Rightarrow y = -3.$ $F(-3,5)$, $F'(-3,-3)$



68) Let P be the population at time t.

$$\frac{dP}{dt} \propto P.$$

$$\frac{dP}{dt} = kP.$$

$$\frac{dP}{P} = k\,dt$$

(Integrating)

$$\int \frac{dP}{P} = \int k\,dt$$

$$\log P = kt + c$$

$$P = e^{kt} e^{c}$$

$$\boxed{P = c e^{kt}}$$

When $t = 0$, $P = 1,30,000$

$$P = C e^{0}$$

$$C = 1,30,000$$

when $t = 1$, $P = 1,60,000$

$$P = 1,30,000\, e^{k}$$

$$e^{k} = \frac{1,60,000}{13,0000}$$

$$e^{k} = \frac{16}{13}$$

$$k = \log \frac{16}{13}$$

$$k = 0.2070$$

10

To find P when $t = 2$ ie (in 2020)

$$P = Ce^{kt}$$

$$P = 1{,}30{,}000\, e^{0.207 \times 2}$$

$$= 1{,}30{,}000\, e^{0.42}$$

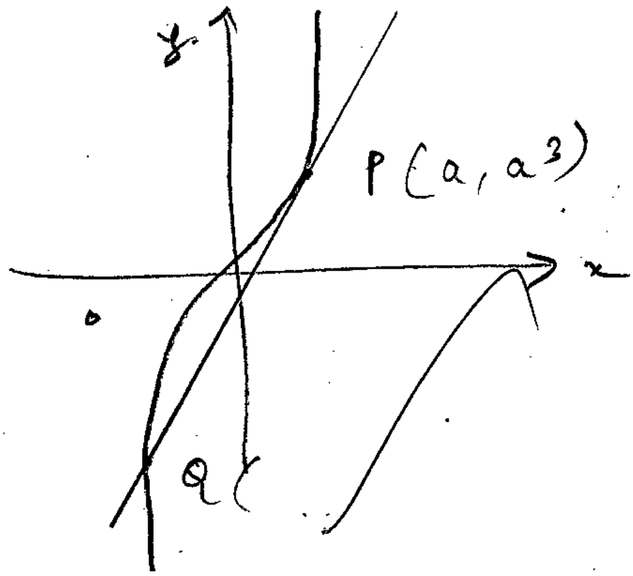
$$= 1{,}30{,}000 \times 1.52$$

$$= 197600$$

By 20,20, the population would be 1,97,600

$$\begin{array}{r} 207 \\ \times 2 \\ \hline 414 \end{array} \qquad \begin{array}{r} 152 \\ \times 13 \\ \hline 456 \\ 152 \\ \hline 1976 \end{array}$$

70 a)



Given $y = x^3$.

Let P be a point at ~~a this~~ the

$y = x^3$ $P(a, a^3)$

Diff $y = x^3$ w r t x :

$$\frac{dy}{dx} = 3x^2.$$

$$\left(\frac{dy}{dx}\right)_{a, a^3} = 3a^2.$$

Eqn of tangent.

$$y - y_1 = m(x - x_1)$$

$$y - a^3 = 3a^2(x - a)$$

Solving $y = x^3$, eqn ~~of~~ tngt.

$$x^3 - a^3 = 3a^2(x - a)$$

$$(x-a)(x^2 + ax + a^2) = 3a^2(x - a)$$

x^2

$$(x-a)(x^2 + ax - 2a^2) = 0$$

$$(x-a)(x-a)(x+2a) = 0.$$

$\Rightarrow$ $x = \underline{\underline{-2a}}$, as $x \neq a$

$x^2 + ax - 2a^2$.

$x^2 + 2ax - ax - 2a^2$

$x(x+2a)$ $(x+$

10

Coordinates of Q $(-2a, -8a^3)$

Slope at Q $= \left(\frac{dy}{dx}\right)_Q$

$= (3x^2)_{-2a, -8a^3}$

$= 3 \times (-2a)^2$

$= 3 \times 4a^2$

$= 12a^2$.

$= 4(3a^2)$

$= 4$(slope of tangent at P)

$\therefore$ Hence proved

PART - B.

42 $A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix}$

$|A| = -1(-15+16) - 2(20-16) - 2(-16+12)$

$= -1 - 8 - 2(-4)$

$= -1 - 8 + 8$

$= -1 \neq 0.$

A is non-singular and invertible

Let A_{ij} be the cofactor of each element in i^{th} row and j^{th} column.

$A_{11} = +(-15+16) = +1$

$A_{12} = -(20-16) = -4$

$A_{13} = +(-16+12) = -4$

$A_{21} = -(10-8) = -2.$

$A_{22} = +(-5+8) = 3.$

$A_{23} = -(4-8) = 4$

6

$$A_{31} = +(8-6) = 2$$

$$A_{32} = -(-4+8) = -4.$$

$$A_{33} = +(3-8) = -5.$$

$$\text{Cof } A = \begin{bmatrix} +1 & -4 & -4 \\ -2 & 3 & 4 \\ 2 & -4 & -5 \end{bmatrix}$$

$$\text{Adj } A = (\text{Cof } A)^T$$

$$= \begin{bmatrix} +1 & -2 & 2 \\ -4 & 3 & -4 \\ -4 & 4 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A.$$

$$= \frac{-1}{1} \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 4 & -4 & 5 \end{bmatrix}$$

$$A^{-1} = A$$

Hence proved.

43) Let $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

$$\vec{a} \times \vec{i} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{vmatrix}$$

$$= \vec{i}(0) - \vec{j}(0) + \vec{k}(\quad)$$

$\vec{a}\cdot(\vec{b}\times\vec{c})$

43) a) $\vec{i} \times (\vec{a} \times \vec{i}) = (\vec{i}\cdot\vec{i})\vec{a} - (\vec{i}\cdot\vec{a})\vec{i}$

$$= \vec{i}\,\vec{a} - \vec{a}\,\vec{i}$$

$$= \vec{a} - \vec{a} =$$

(ii)

44) Sphere: $C(1, 2, 3)$

$$r = \sqrt{(5-1)^2 + (5-2)^2 + (3-3)^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= \sqrt{16+9} = 5 \text{ sq units}$$

2

Vector eqn:

$$|\vec{r} - \vec{c}| = |\vec{a}|$$

$$|\vec{r} - (\vec{i} + 2\vec{j} + 3\vec{k})| = 5$$

2

5

Cartesian eqn :

$$x\vec{i} + y\vec{j} + z\vec{k} - (\vec{i} + 2\vec{j} + 3\vec{k}) = 5$$

$$(x-1)\vec{i} + (y-2)\vec{j} + (z-3)\vec{k} = 5$$

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = 25$$

4b) Triangle inequality :

The moduli of sum of two complex numbers is less than or equal to to the sum of the moduli.

$$|z_1 + z_2| \leq |z_1| + |z_2|$$

Proof :

$$|z_1 + z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)}$$

$$= (z_1 + z_2)(\overline{z_1} + \overline{z_2})$$

$$= z_1\overline{z_1} + z_1\overline{z_2} + z_2\overline{z_1} + z_2\overline{z_2}$$

$$= |z_1|^2 + |z_2|^2 + z_1\overline{z_2} + z_2\overline{z_1}$$

$|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + \overline{\bar{z}_1 z_2} + z_2\bar{z}_1$

$= |z_1|^2 + |z_2|^2 + 2\,Re(\bar{z}_1 z_2)$

$\leq |z_2|^2 + |z_2|^2 + 2|\bar{z}_1|\,|z_2|$

$(Re\, z \leq |z|)$

$= |z_1|^2 + |z_2|^2 + 2|z_1|\,|z_2|$

$|z_1+z_2|^2 = (|z_1| + |z_2|)^2$

Taking +ve sq roots

$|z_1+z_2| = |z_1| + |z_2|$

Hence proved.

47) $\lim_{x\to 0} x^2 \log e^x$

$\lim_{x\to 0} \frac{\log e^x}{1/x^2} = \left(\frac{\infty}{\infty}\right)$ I F

$\frac{0}{\infty}$

$\frac{x}{1/x^2}$

(By L'Hopital)

$\frac{\infty}{\infty}$

12

$$\lim_{x \to 0} \frac{\frac{e^x}{e^x}}{-\frac{2}{x^3}}$$

x^{-2}

$-2x^{-3}$

$$= \lim_{x \to 0} \frac{x^3}{-2}$$

$$\lim_{x \to 0} \frac{x^3}{-2} = \frac{0}{-2} = 0$$

4a) $\int_0^1 \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}} \, dx$

Let $t = \sin^{-1} x$

$\int_0^1 \cancel{t^3}$

$$dt = \frac{1}{\sqrt{1-x^2}} \, dx$$

x	0	1
t	0	$\pi/2$

$$= \int_0^{\pi/2} t^3 \, dt$$

$$= \left[\frac{t^4}{4}\right]_0^{\pi/2} = \left(\frac{\pi}{2}\right)^4 = \frac{\pi^4}{4 \times 16} = \frac{\pi^4}{64}$$

$$= \frac{\pi}{64}$$

50) $(2D^2+5D+2)y = e^{-\frac{1}{2}x}$

~~Parti~~ Characteristic eqn:

$$2p^2+5p+2=0$$

$$2p^2+4p+p+2=0$$

$$2p(p+2)+1(p+2)=0$$

$$(2p+1)(p+2)=0$$

$$p=-\tfrac{1}{2}, \quad p=-2$$

$$C.F = Ae^{-\frac{1}{2}x} + Be^{-2x}$$

P.I:

$$\frac{1}{2D^2+5D+2} e^{-\frac{1}{2}x}$$

$$= \frac{1}{(2D+1)(D+2)} e^{-\frac{1}{2}x}$$

$$= \frac{1}{(2D+1)\left(-\frac{1}{2}+2\right)} e^{-\frac{1}{2}x}$$

$$= \frac{2}{(2D+1)\,3} e^{-\frac{1}{2}x}$$

$2-\frac{1}{2}$

$\frac{4-1}{2}$

$\frac{3}{2}$

$$= \frac{2x\, e^{-\frac{1}{2}x^2}}{3}$$

$$y = C \cdot F + P \cdot I$$

$$= Ae^{-\frac{1}{2}x} + Be^{-2x} + \frac{2x e^{-\frac{1}{2}x^2}}{3}$$

51) $p \Leftrightarrow q$

LHS :

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

RHS :

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$\sim q \vee p$	$(\sim p \vee q) \wedge (\sim q) \vee p$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

From the above two tables. the last column is identical.

Hence proved.

$p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

52) Cancellation law :

Let G be a group.

Let $a, b \in G$.

If $a * b = a * c$ then $b = c$ (By left Cancellation law)

$b * a = c * a$ then $b = c$ (By right Cancellation law)

Proof.

1) $a * b = a * c$.

$a^{-1} * (a * b) = a^{-1} * (a * c)$

$(a^{-1} * a) * b = (a^{-1} * a) * c$ (By identity axiom)

$e * b = e * c$. (By inverse axiom)

$\boxed{b = c}$

2) $b * a = c * a$

$(b * a) * a^{-1} = (c * a) * a^{-1}$

$b * (a * a^{-1}) = c * (a * a^{-1})$ (By inverse axiom)

$b * e = c * e$. (By identity axiom)

$\boxed{b = c}$

Hence proved.

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கூடுதல் விடைத்தாள்

55) பதிவு எண்....................

5) a)

c)

Eqn of asymptote.

$3x^2-5xy-2y^2+17x+y+14=0$.

is the eqn of hyperbola.

Comparing with pair of lines.

$(ax^2+2hxy+by^2+2gx+2fy+c=0$.

$a=3$, $h=-5/2$, $b=-2$.

~~Angle~~,

The equation of asymptotes differ from hyperbola by a constant term

∴ ~~Eqn of hyperbola~~ Combined equation of asymptotes

$3x^2-5xy-2y^2+17x+y+k=0$.

Angle between the asymptotes =

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$$

$$= \frac{2\sqrt{\frac{25}{4}-3\times-2}}{3+(-2)}$$

6

$$\tan\theta = \frac{2\sqrt{\frac{25}{4}+6}}{3+2}$$

$\frac{24}{4}$

$\frac{24}{\frac{25}{49}}$

$$= \frac{2\sqrt{\frac{49}{4}}}{7}$$

$$= \cancel{2}\,\frac{7}{\cancel{2}}$$

$$\tan\theta = 7$$

$$\boxed{\theta = \tan^{-1}(7)}$$

43) (i) $\vec{i} \times (\vec{a} \times \vec{i})$

$$\vec{i} \times (\vec{a} \times \vec{i}) = (\vec{i} \cdot \vec{i})\,\vec{a} - (\vec{a} \cdot \vec{i})\,\vec{i}$$

$$= \vec{a} - (a_1\vec{i} + a_2\vec{j} + a_3\vec{k} \cdot \vec{i})$$

$$= \vec{a} - (a_1 \cdot \vec{i})\,\vec{i}$$

$$= \vec{a} - a_1\vec{i} \rightarrow 1$$

Similarly

$\vec{j} \times (\vec{a} \times \vec{j}) = \vec{a} - a_2\vec{j}$ (2)

$\vec{k} \times (\vec{a} \times \vec{k}) = \vec{a} - a_3\vec{k}$. (3)

(1) + (2) + (3)

$\therefore$ $\vec{i}$.

$\vec{i} \times (\vec{a} \times \vec{i}) + \vec{j} \times (\vec{a} \times \vec{j}) + \vec{k} \times (\vec{a} \times \vec{k})$

$= 3\vec{a} - (a_1\vec{i} + a_2\vec{j} + a_3\vec{k})$

$= 3\vec{a} - \vec{a}$

$= 2\vec{a}$.

Hence proved.

(ii) $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-4}{6}$

$\vec{u} = 2\vec{i} + 3\vec{j} + 6\vec{k}$. $\longrightarrow$ (1).

$\dfrac{x+1}{1} = \dfrac{y+2}{2} = \dfrac{z-4}{2}$.

$\vec{v} = \vec{i} + 2\vec{j} + 2\vec{k}$.

6

Angle between the lines

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|}$$

$$\cos\theta = \frac{2+6+12}{\sqrt{4+9+36}\,\sqrt{1+4+4}}$$

$$= \frac{20}{\sqrt{49}\,\sqrt{9}}$$

$$\cos\theta = \frac{20}{7 \times 3}$$

$$\theta = \cos^{-1}\left(\frac{20}{21}\right)$$

S.S.L.C. 3-30,00,000-2013-G.C.P., Pu.T.

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கூடுதல் விடைத்தாள்

பதிவு எண்.....................